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$$=r\sin\theta+\sqrt{[r^2\sin^2\theta-2rx(1+\cos\theta)]}.$$

$$\therefore\ x=\frac{2r\,[(\sin\theta-\sin\varphi)\sin(\theta+\varphi)-(\sin\theta-\sin\varphi)^2]}{(\cos\theta+\cos\varphi)^2}$$

$$=2r[\sin\tfrac{1}{2}(\theta+\varphi)\sin\tfrac{1}{2}(\theta-\varphi)\sec^2\tfrac{1}{2}(\theta-\varphi)-\tan^2\tfrac{1}{2}(\theta-\varphi)].$$

$$\pi x^2=4\pi r^2[\sin\tfrac{1}{2}(\theta+\varphi)\sin\tfrac{1}{2}(\theta-\varphi)\sec^2\tfrac{1}{2}(\theta-\varphi)-\tan^2\tfrac{1}{2}(\theta-\varphi)]^2.$$

Let L=average length, $\triangle$ =average area.

$$\therefore\ L=\frac{\int_0^\pi\int_0^\theta x\,d\theta d\varphi}{\int_0^\pi\int_0^\theta d\theta d\varphi}=\frac{2}{\pi^2}\int_0^\pi\int_0^\theta x\,d\theta d\varphi$$

$$=\frac{8r}{\pi^2}\int_0^\pi(\theta\cos^2\tfrac{1}{2}\theta-\sin\theta+\sin\theta\log\sec\tfrac{1}{2}\theta)d\theta=\frac{2r}{\pi^2}(\pi^2-8)=.3789r.$$

$$\triangle=\frac{\pi\int_0^\pi\int_0^\theta x^2d\theta d\varphi}{\int_0^\pi\int_0^\theta d\theta d\varphi}=\frac{2}{\pi}\int_0^\pi\int_0^\theta x^2d\theta d\varphi$$

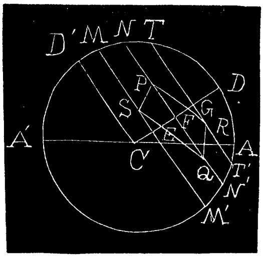
$$=\frac{32r^2}{3\pi}\int_0^\pi(6\theta\cos^4\tfrac{1}{2}\theta-3\theta\cos^2\tfrac{1}{2}\theta-6\sin\tfrac{1}{2}\theta\cos^3\tfrac{1}{2}\theta+2\sin^3\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta$$

$$-12\sin\tfrac{1}{2}\theta\cos^3\tfrac{1}{2}\theta\log\cos\tfrac{1}{2}\theta)d\theta=\frac{4r^2}{3\pi}(3\pi^2-28)=.2174\pi r^2.$$

85. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa.

Two points are taken at random in a circle and a chord drawn through them; a point is then taken at random in each segment. Find the average area of the quadrilateral formed by joining the four points.

Solution by the PROPOSER.



Let P, Q, R, S be the four random points taken as indicated in the problem, NN' the chord through PQ, MM', TT' the chords through S, R, respectively.

Draw CD perpendicular and CD' parallel to NN'. Let $CD=r$, $CE=u$, $CF=v$, $CG=w$, $NQ=x$, $PQ=y$, $NF=\sqrt{(r^2-v^2)}=z$, $TG=\sqrt{(r^2-w^2)}=t$, $ME=\sqrt{(r^2-u^2)}=s$, $\angle D'CA'=\theta$.

An element of the circle at P is $dvdx$; at Q, $yd\theta dy$; at R, $2tdw$; at S, $2sdu$.

The limits of θ are 0 and $\frac{1}{2}\pi$ and doubled; of v, $+r$ and $-r$; of x, $2z$ and 0; of y, 0 and x and doubled; of w, v and r and doubled; of u, $-r$ and v and doubled. Area $PRQS=\frac{1}{2}y(w-u)$.

The whole number of ways four points can be taken in the circle is π^4r^8.

$$\therefore \triangle=\frac{16}{\pi^4r^8}\int_0^{\frac{1}{2}\pi}\int_{-r}^{r}\int_0^{2z}\int_0^{x}\int_v^{r}\int_{-r}^{v}\tfrac{1}{2}y(w-u)d\theta dvdxydy2tdw2sdu$$

$$=\frac{8}{3\pi^4r^8}\int_0^{\frac{1}{2}\pi}\int_{-r}^{r}\int_0^{2z}\int_0^{x}\int_v^{r}[4(a^2-v^2)^{\frac{3}{2}}+6vw\sqrt{(a^2-v^2)}+6a^2w\sin^{-1}(v/a)$$

$$+3\pi a^2w]ty^2d\theta dvdxdydw$$

$$=\frac{16}{3\pi^3r^6}\int_0^{\frac{1}{2}\pi}\int_{-r}^{r}\int_0^{2z}\int_0^{x}(a^2-v^2)^{\frac{3}{2}}y^2d\theta dvdxdy$$

$$=\frac{16}{9\pi^3r^6}\int_0^{\frac{1}{2}\pi}\int_{-r}^{r}\int_0^{2z}(a^2-v^2)^{\frac{3}{2}}x^3d\theta dvdx$$

$$=\frac{64}{9\pi^3r^6}\int_0^{\frac{1}{2}\pi}\int_{-r}^{r}(a^2-v^2)^{\frac{7}{2}}d\theta dv=\frac{35r^2}{18\pi^2}\int_0^{\frac{1}{2}\pi}d\theta=\frac{35r^2}{36\pi}.$$

MISCELLANEOUS.

74. Proposed by S. HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

The longest diameter of a horizontal ellipse is $CB=2a=6$ feet. Its shortest diameter is $EF=2b=4$ feet, their intersection being at D. Find in an indefinite vertical plane passing through CB, a point $A=5$ feet$=c$ from D, the ellipse being seen from A as a circle.

III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

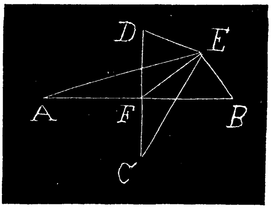
To find a point E in the vertical plane AEB at which DC, AB subtend the same angle, we proceed as follows: Let $DF=FC=b=2$, $AF=FB=a=3$, $FE=c=5$, $AE=x$, $BE=y$, $DE=CE=z$, $\angle DEC=\angle AEB=\theta$.

Then $c=\frac{1}{2}\sqrt{(2x^2+2y^2-4a^2)}=\frac{1}{2}\sqrt{(x^2+y^2+2xy\cos\theta)}$.

$\therefore 2(a^2+c^2)=x^2+y^2$; $4c^2=x^2+y^2+2xy\cos\theta$.

$\therefore x^2+y^2=2a^2+2c^2$ ..... (1).

$2xy\cos\theta=2c^2-2a^2$ ..... (2).



But $c=\frac{1}{2}\sqrt{(4z^2-4b^2)}=\frac{1}{2}\sqrt{(2z^2+2z^2\cos\theta)}$.

$\therefore z=\sqrt{(b^2+c^2)}$ and $\cos\theta=\frac{c^2-b^2}{c^2+b^2}$.

$$\therefore 2xy=\frac{2(c^2-a^2)(c^2+b^2)}{c^2-b^2}.\quad \therefore x+y=\frac{2\sqrt{(c^4-a^2b^2)}}{\sqrt{(c^2-b^2)}},\ x-y=\frac{2c\sqrt{(a^2-b^2)}}{\sqrt{(c^2-b^2)}}.$$